

1. If an assignment is not neat and formatted as shown in the attached example, the assignment will be counted as not having been done.
 - a. In order to attain the degree of clarity and neatness required, you may need to work certain problems first on scrap paper then record your final work on your homework paper.
2. If you are unable to work a particular problem, you must for that problem
 - a. write at least one *specific* question whose answer you believe would help you to solve that problem, and
 - b. explain what you did do in order to learn from the problem while doing the assignment.

We will discuss in class specifically what is intended in #2 above.

Please show and discuss this page and the attached example to a parent and ask parent to sign below. Return this with signature on Monday, October 19.

Parent signature

Example of properly completed and formatted homework

10/13/08
 [HH] 14G #8(c-i), 9a,
 10a, 11, 12, 13, 14a
 J11 pp 91-94 #1-4

HW

P322 #8 | all matrices are 2×2 , I : identity matrix, simplify:

c. $A(A^2 - 2A + I)$

$= A^3 - 2A^2 + AI$

$= A^3 - 2A^2 + A$ { $AI = IA = A$ }

d. $A(A^2 + A - 2I)$

$= A^3 + A^2 - 2AI$

$= A^3 + A^2 - 2A$

e. $(A+B)(C+D)$

$= (A+B)C + (A+B)D$

$= AC + BC + AD + BD$

f. $(A+B)^2$

$= (A+B)(A+B)$

$= (A+B)A + (A+B)B$

$= A^2 + BA + AB + B^2$ { $AA = A^2, B \cdot B = B^2$ }

g. $(A+B)(A-B)$

$= (A+B)A + (A+B)(-B)$

$= A^2 + BA - AB - B^2$

h. $(A+I)^2$

$= (A+I)(A+I)$

$= (A+I)A + (A+I)I$

$= A^2 + IA + AI + I^2$

$= A^2 + A + A + I$ { $I^2 = I$ }

$= A^2 + 2A + I$

i. $(3I-B)^2$

$= (3I-B)(3I-B)$

$= (3I-B)3I + (3I-B)(-B)$

$= 9I^2 - 3BI - 3IB + B^2$

$= 9I - 3B - 3B + B^2$

$= 9I - 6B + B^2$

P323 #9 | a. If $A^2 = 2A - I$, find A^3 and A^4 in linear form, $kA + lI$

$A^3 = A \times A^2$

$= A(2A - I)$

$= 2A^2 - AI$

$= 2(2A - I) - AI$

$= 4A - 2I - AI$

$= 4A - 2I - A$

$= 3A - 2I$

$A^4 = A \times A^3$

$= A(3A - 2I)$

$= 3A^2 - 2AI$

$= 3(2A - I) - 2A$

$= 6A - 3I - 2A$

$= 4A - 3I$

#10 | a. If $A^2 = I$, simplify:

i. $A(A+2I)$

$= A^2 + 2AI$

$= I + 2A$

ii. $(A-I)^2$

$= (A-I)(A-I)$

$= (A-I)A + (A-I)(-I)$

$= A^2 - IA - AI + I^2$

$= I - A - A + I$

$= 2I - 2A$

iii. $A(A+3I)^2$

$= A(A+3I)(A+3I)$

$= (A^2 + 3AI)(A+3I)$

$= (A^2 + 3A)A + (A^2 + 3A)3I$

$= A^3 + 3A^2 + 3A^2I + 9AI$

$= A + 3I + 3I^2 + 9A$

IA $= 10A + 6I$

#11) The result "if $ab=0$, then $a=0$ or $b=0$ " for real #s does not have an equivalent result for matrices.

a. If $A = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$.

Find AB .

$$AB = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = 0$$

This example provides us with evidence that "If $AB=0$ then $A=0$ or $B=0$ " is a false statement.

b. If $A = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} \end{bmatrix}$, determine A^2 .

$$A^2 = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} \end{bmatrix} \begin{bmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} \end{bmatrix}$$

c. Comment on the following argument for a 2×2 matrix A :

It is known that $A^2=A$. $\therefore A^2-A=0$

$$\therefore A(A-I)=0$$

$$\therefore A=0 \text{ or } A-I=0$$

$$\therefore A=0 \text{ or } A=I$$

Because of 'a', this is not necessarily true.

d. Find all 2×2 matrices A for which $A^2=A$.

~ (Hint: Let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$)

① $I^2=I$ ② zero matrix

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} a^2+bc & ab+bd \\ ac+cd & bc+d^2 \end{bmatrix}$$

$$A^2=A \Leftrightarrow \begin{bmatrix} a^2+bc & ab+bd \\ ac+cd & bc+d^2 \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$$c = \frac{a-a^2}{b}$$

$$\leftarrow 1) a^2+bc = a$$

$$a+d=1$$

$$b = ab+bd \leftarrow 2) ab+bd = b \quad b(a+d) = b \quad a+d=1$$

$$\boxed{d=1-a}$$

$$= ab+b(1-a) \quad 3) ac+cd = c \quad c(a+d) = c \quad a+d=1$$

$$a=1-d$$

$$= ab+b-ab \quad 4) bc+d^2 = d$$

$$= 1-1+a$$

$$\boxed{b=b}$$

$$\boxed{a=a}$$

$$\textcircled{3} \begin{bmatrix} a & b \\ a-a^2 & 1-a \\ b & 1-a \end{bmatrix}$$